Neural modeling of sequential inferences and learning over episodic memory

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Abstract

Episodic memory is a significant part of cognition for reasoning and decision making. Retrieval in episodic memory depends on the order relationships of memory items which provides flexibility in reasoning and inferences regarding sequential relations for spatio-temporal domain. However, it is still unclear how they are encoded and how they differ from representations in other types of memory like semantic or procedural memory. This paper presents a neural model of sequential representation and inferences on episodic memory. It contrasts with the common views on sequential representation in neural networks that instead of maintaining transitions between events to represent sequences, they are represented as patterns of activation profiles wherein similarity matching operations support inferences and reasoning. Using an extension of multi-channel multi-layered adaptive resonance theory (ART) network, it is shown how episodic memory can be formed and learnt so that the memory performance becomes dependent on the order and the interchange of memory cues. We present experiments as a proof of concepts to show that the model contrasts sequential representations in semantic memory with those in episodic memory and the model can exhibit transitive inferences consistent with human and animals data.

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1. Introduction

Episodic memory is a particular type of long-term declarative memory that stores specific past experiences. In contrast, the other type of declarative long-term memory is semantic memory which stores general facts, concepts, or rules. Episodic memory plays a key role in binding many aspects of human everyday lives. It provides the extension of perceptions beyond momentary perceptual ranges by providing past relevant episodes. Studies in neuropsychology have shown that damages in episodic memory caused by lesions in hippocampal areas in human and animals impair significant cognitive capacities [1–3]. Most of these studies agreed that episodic memory and hippocampus deal with declarative retrieval of the past specific experienced events apart from other non-declarative memory. Without the episodic memory, a situation cannot be properly referred to in its past context. This kind of cognitive deficit resulting from episodic memory impairment indicates the main feature of hippocampus or episodic memory for rapidly storing and binding daily events for latter use [4].

Various computational models have been developed to explore different aspects of episodic memory. Many contemporary models suggest that the hippocampus works as conjunctions of specific patterns coming from different stimuli [5–8] before they are recalled for later use or consolidated to more permanent forms. Various architectures and approaches have been proposed to model the sequential representation for episodic memory using statistical models (e.g. [9]), connectionist architectures (e.g. [10,11,5]), symbolic models (e.g. [12–14]), and probabilistic matching (e.g. [15]). These models have demonstrated that sequential structure of episodes can be represented, stored, and recalled later based on memory cues. Most of these models assume a standard functionality of episodic memory that a sequence of events can be learnt rapidly at once and recalled later based on a memory cue consisting of a subsequent presentation of events partially of the original sequence. This assumption implies that hippocampus, as the main part of episodic memory in the brain, only performs pattern completion to recollect or predict the complete episode based on the first few subsequent steps of the episode. It has been suggested that hippocampus should interact with its surrounding cortical areas in order to realize the complete functionality of episodic memory [8].

However, beyond memory recalls, there are evidences showing that hippocampus handles different temporal contexts of memory tasks, cues, and entries supporting further processes of decision making, reasoning, and learning in general. The accuracy of retrieval
incidentally depends on the relative position of cues, lags between memory items, and the context from previous operations [16–21] allowing it to construct novel patterns and handle noisy, corrupted, or low fidelity memory cues.

Understanding how does episodic memory work, the structure of information stored, its distinctions and relationships with other types of memory (e.g. semantic or procedural memory) can provide useful insight for building an intelligent computational agent that autonomously explore and learn from the environment. Aside from its value to gain more knowledge about human memory, the model of inference and reasoning with episodic memory may guide the design towards more efficient reasoning and decision making of an intelligent agent architecture.

In this paper, a model of episodic memory emphasizing its support in inferences and decision making is presented. The model comprises the long-term memory buffer resembling hippocampal systems. In contrast to other more realistic models of episodic memory, the proposed model simplifies the neural structure of hippocampus but caters different memory functionalities supporting inferences and learning. The model is made as a multi-layered and multi-channel Adaptive Resonance Theory (ART) neural network. However, to handle temporal or sequential order, it relies on a special encoding wherein temporal events are captured as analog patterns both in neural activations and synaptic weights. This emphasizes that a memory item is stored with a magnitude level reflecting its order relative to other items in the sequence. This allows event instances, memory cues, and sequential constraints to be expressed and manipulated through pattern matching and completion operations. This special neural network model also supports dynamic adjustment of matching parameters enabling explicit indications of its recognition level and similarity of the retrieved item. Moreover, the model inherently supports incremental learning in which neural codes for events and episodes can be dynamically allocated on demand based on the novelty in the incoming memory entries. This last feature provides an advantage over other sequential memory models based on sequential chaining inference models like those with probabilistic or recurrent auto-associative methods.

This paper provides theoretical accounts of the structure of episodic memory and its supported operations. The characteristics of sequential representation and processing in hippocampus are contrasted with the ones in other kinds of memory like semantic and procedural that may rely on probabilistic chaining mechanisms. Case studies are shown to confirm the characteristics of episodic memory to support inferences in comparison to semantic/procedural memory. The case studies investigate the use of episodic memory for transitive inferences and temporal context formation for learning and reasoning. The results show that the characteristics of the proposed model are consistent with human and animal data.

The rest of the paper is structured as follows. The next section provides an overview of studies on episodic memory and hippocampus. The next section reviews related works of modelling episodic memory. Section 3 describes the proposed episodic memory model. Section 4 describes the implementation and case studies of the proposed model to investigate the transitive inference and contextual alternation learning capabilities.

2. Related work

Studies with computational models have revealed particular roles of episodic memory in supporting cognitive functions. The models are made to study different aspects of memory according to the interests and research objectives. Different aspects of memory like scalability [10], performance [11,22], and similarity with human memory performance in weighing memory traces [23–25]9 have been the main concerns.

In the context of computational agents in virtual environment, the survivability of a situated agent can be improved since a wide range of cognitive functions from sensing, reasoning, and learning is supported by episodic memory [12,13]. The incorporation of episodic memory may also add the realism of an artificial character. As a source of autobiographical record, episodic memory provides the necessary information about past experiences that can be useful to interact with the environment and communicate with other agents [26–28]. However, most of the aforementioned models still assume a limited set of memory operations. The common process consists of reconstructing the complete memorized episode, given a cue as a partial representation of a state or an event snapshot. The recalled sequence may be produced as a series of readout starting from the first cue presentation. For example, the episodic memory extension of SOAR cognitive architecture [13] uses a simple cue-based retrieval to recollect an episode and the complete sequence can be reconstructed by retrieving the next (or previous) element of the episode one after another in a subsequent order.

Other works on computational model have looked at possible neural structures wherein episodic memory is resided in the brain. Gluck and Myers suggest the neural network structure of episodic memory and more-permanent semantic memory showing how both interact to robustly learn and store information [29]. A similar but more complete model covers the memory consolidation process from episodic memory to more permanent abstract form in semantic memory [6]. The model suggests that episodic and semantic memory work complementarily side by side and learn independently in different rates. Episodic memory captures specific patterns rapidly in a single pass while semantic memory learns general facts and abstract knowledge in a slow and gradual manner. The model explains how complex and abstract knowledge can be formed in semantic memory based on the specific experiences captured in episodic memory.

Similarly, O’Reilly and Rudy argue that the episodic memory functionality can be supported by the interaction between hippocampus and some surrounding cortical areas [8]. They suggest that solving conjunctive learning tasks does not totally require hippocampus but can be partially supported by some cortical areas. The complementary model of cortical/hippocampal memory system puts aside the idea of hippocampus as a part of declarative memory and suggests that the hippocampus is only important for incidental tasks. Although the model considers the contextual relations to be substantiated as recurrent connections in the hippocampal CA3 area, it is still assumed that the sequential relations in cognitive tasks are provided explicitly as additional inputs to hippocampus.

The cortical/hippocampal interaction model can explain the associative distance effects [17] in which the performance of accurately retrieving the correct memory entries depends on the lags between cue items wherein other monolithic neural architectures still fail to explain this phenomena. It is suggested that the distance effect is actually influenced by the cortical parts of the memory system that learn to select the stimulus presented as the input to the hippocampus. Another work look at the sequential relations between items and contexts for retrieval in hippocampus. Based on probabilistic model called TRBM (Temporal Restricted Boltzman Machine), hippocampus is modeled consisting of different components according to its anatomical structure in the brain [30]. The current state of the world is represented as a unitary form in the symmetric auto-associative network. The unitary representation enables complex description of momentary condition in a single state. To represent the sequential structure, explicit transitions link states that occur subsequently. This model

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supports retrieval of complex sequential patterns by calculating the probability distribution of every possible episode following the memory cues presentation. As a probabilistic model of sequential memory, it involves recursive processes or value iterations to calculate the probability distribution over each possible matching episode. However, it is still unclear how scalable it is to deal with a large number of episodes. The model does not specify how the probability distribution can be learnt continuously online.

Another model called TCM (Temporal Context Model) has been made to model hippocampus based on long-term recency and contingency effects in hippocampus using a recurrent neural networks [15]. This model accurately emulates the recency effects in hippocampus by storing contextual information relating one event to another through a continual process called contextual drift. Similar to the TRBM model, this model does not specify exactly how the contextual drift should be performed. There are alternatives mechanisms of contextual drift that can be applied according to the kind of processes and characteristics of episodic memory to be evaluated.

Both probabilistic and recurrent models (e.g. TRBM, TCM) may emulate the characteristics of simple operations and reasoning in hippocampus like recency effects and linearly ordered traversal of events in a sequence. The probabilistic model may also be able to predict the complete sequence of the episode given a few subsequent events in the beginning. However, these models do not explain how more complex processes and reasoning in episodic memory can be conducted like transitive inference, sequence disambiguation, planning, prediction and so on which have been observed in humans and animals in the context of episodic memory tasks [19,16,20,31–33]. Recent findings in neuroscience have shown that patterns of neural activities in hippocampus reflect temporal positions of objects in a sequence [34] and can retrieve novel paths or trajectories towards novel goals [35]. Hierarchical Transition Memory (HTM) is another type of neural network in which sequences nodes arranged in hierarchical structure [36]. The network memorizes frequently observed patterns that occur at the same time. It also links nodes that are activated subsequently. Similar to the other recurrent or associative networks (e.g. TRBM or TCM), it recognizes sequential patterns based on direct transition links between nodes.

Different kinds of inference mechanisms in hippocampus have also been considered to be the primary support for cognitive functionality. A special type of neural networks has been devised to model hippocampus using sparsely recurrent connections [5]. The recurrent network structure enables recall and prediction of sensory sequences based on the presentation of a partial sequence distorted or changed to some extent from the original target episode. In this way, the memory cues can become the criteria to infer or reason over hippocampus besides as cues for retrieval. This model supports multiple features of inferences like spontaneous replay, transverse patterning, sequential disambiguation, and transitive inference [37,38] in a single hippocampal system. However, this model does not explain how inference can actually be conducted over the memory particularly when it interacts with other cortical areas in the brain. It does not have a clear mechanism or indicator to determine the start (or end) of an episode in recognition and/or learning processes. Moreover, since the recurrent associations in the network are usually assumed to have equal underlying strengths, the model cannot explain the distance effects between memory items during inferences.

In this paper, it is suggested that the relationship between memory items can be represented as associative strength that are gradually increasing (or decreasing) following their temporal (order) positions in the episode. It contrasts with recurrent or probabilistic structure considered to be more pervasive in other parts of cortical areas (e.g. semantic memory, procedural memory). Some previous works suggested that this kind of gradual values profile is more robust in retrieving sequential patterns than the probabilistic model like HMM (Hidden Markov Model) when memory cues are severely altered or corrupted [39]. This can happen because calculating the probability distribution over possible episodes requires a recursive or iterative algorithm that is sensitive to the exact order of the sequence. On the other hand, our model in [39] treats the whole chunk of the sequence as an activation profile or a real vector in which a similarity matching can be directly applied. In this way, reasoning or inferences can be realized as a similarity matching as well in which the cues are formed as a list of items in an order reflecting temporal constraints of the inference.

This kind of inference over memory has been applied as a memory consolidation mechanism to generate semantic knowledge about objects in a real-time video game environment [40] and as a hierarchical planning process [41]. The sequential activation profile in forming episodic memory and similarity matching among the profiles allows learning and retrieval to be conducted rapidly which is also evident in hippocampus and episodic memory tasks. Another important distinction of this model is its incremental nature that new neurons (nodes) and connections may be allocated at runtime during learning or retrieval allowing the network to grow. This feature is incorporated following the contemporary views of neurogenesis and plasticity in neuroscience. Contrary to the long-standing view in the past that neurons can significantly regenerate only during developmental stage and hardly occur in adult brains, it has been evident nowadays that neural regeneration and plasticity occur in adult brains as well in certain areas and for particular contexts, especially in hippocampus. A large amount of neural granule cells in DG (dentate gyrus) area of hippocampus is generated and retained everyday [42] and functionally integrated to the neural circuits of hippocampus [43]. In this paper, it is viewed that the generation and plasticity of neurons and their connections are the key aspects that enable continual and incremental learning in episodic memory. These growing processes may also be complementarily paired with neural pruning or forgetting. Our former works in this model include the study of balancing online learning and forgetting in the same model of episodic memory [40,39]. This paper, however, focuses on inference supports in the same type of episodic memory complementary to another kind of memory in cortical areas.

3. Modelling episodic memory

It has been commonly known that hippocampus is the particular part of the brain serving most of the functionality of episodic memory. Anatomically, hippocampus has a distinct structure compared to other parts in the brain [44]. Many parts make hippocampus. The most extensively studied areas on their functionality are CA1 and CA3 (see Fig. 1). Both are interconnected to each other and to Entorhinal Cortex (EC) as the bulk of input (output) from (to) cortical areas. EC also projects to dentate gyrus (DG) which is sparsely connected to CA3 via the particular mossy synaptic fibers. These mossy fibers connected to granule cells which are particular neural cells that can be relearn dynamically in response to demands of memory space. These features allow hippocampus to learn and capture information rapidly in a simple pass of pattern presentations rather than gradually acquire knowledge through repetitive trials.

Studies with human and animal subjects indicate that hippocampus accommodates different types of inferences and reasoning tasks involving memory operations [19,16,20,31–33]. The nature of the tasks includes temporal, spatial, and/or contextual (e.g. odor discrimination, object novelty recognition, symbolic preferences). All those tasks involve sequential representation and processing. It is also evident that the sequences encoded in hippocampus have
the tendency towards a certain temporal direction (forward to the future rather than back to the past) instead of equally preferred [21]. Interestingly, retrieval in logical tasks (e.g. transitive inference) can also be incidental towards a certain direction and dependent on the lag between memorized items in the sequence [17]. For example, in a transitive inference task, people usually perform better in inferring patterns that learnt more distant away in the sequence rather than the closer ones.

It can be considered that episodic memory maintains information about experienced events including their sequential order. It stores not just the sequences of events but also relative association strengths reflecting the order between event items which is evident in [34]. The stored episodes can be retrieved back to the cortical areas in the brain based on memory cues reflecting the items to be retrieved and their sequential order. The retrieval tasks with the sequential cues comprise recognition and recall.

Retrieval (and learning) sequential patterns are not just the properties of episodic memory alone. Other types of memory resided in cortical areas like semantic or procedural memory may also store and retrieve sequential information based on cues as partial patterns of the target entry. However, the common view in neural modeling of sequential memory still consider probabilistic relations or recurrent connections between direct subsequent memory items, like in [30,15]. In this case, each item in memory is only associated with the next direct subsequent item in the sequence. To retrieve the complete sequence, an iterative or recursive processing is required to traverse the item one at a time.

With probabilistic structure, recalling the entire sequence may require calculation of conditional probabilities for every step of action in the sequence.

On the other hand, in this paper, episodic memory is regarded as maintaining the relative strengths to represent the order in the sequence as a whole chunk of the episode. It is hypothesized that episodic memory maintains specific events and sequences of them as units of episode so that, later, it can retrieve or activate the whole sequence at once which enables rapid traversal in the episode. In other words, episodic memory continually develops maps of spatial, temporal, and contextual situations so that rapid recall of a sequence or trajectory is possible for further processing in working memory.

In this paper, the focus is on modelling the episodic memory computationally to serve the main functionality of maintaining episodes as sequences of experienced events. The model supports the sequential recall and recognition operations but also emulating the tendency towards a certain direction in the learnt sequence and the effect of relative position of the target memory items in the learnt sequence. We also view that the episodic memory or hippocampal system provides implicit and explicit mechanisms for both memory encoding and retrieval supporting cognitive functions.

The proposed episodic memory model is considered to be an integral part of the entire reasoning system. At one point of time a snapshot of an individual perception and some information characterizing a single experience can be encoded and held on entorhinal cortex (EC) as a pattern of event to be stored in episodic memory. The information held in EC may include spatial location, orientation, time point, perceived objects, and others related to the context. Episodic memory automatically stores and organizes the events in a sequential order into cognitive units of episodes to be retrieved later when needed. In what follows, the aspects above are formulated and realized in a neural network architecture.

3.1. Issues in general model of episodic memory

Episodic memory receives event as a single unit of input. A single event consists of a number of attributes reflecting the condition or situation in the environment and perception of the agent. An event can be defined as a tuple of attributes.

Definition 1. An event $e$ is a tuple reflecting a moment of experience such that $e = (e_1, e_2, \ldots, e_n)$. Each attribute $e_i$ is defined as a tuple such that $e_i = (e_i^1, e_i^2, \ldots, e_i^q)$ and $q_i$ is a normalized real value $0 \leq q_i \leq 1$.

The received $e$ may be stored as a new entry in episodic memory. To retrieve a memory entry, a memory cue can be formed and input as a criterion to retrieve the entry. A memory cue of event can be expressed as an event sub-pattern.

Definition 2. Let $q = (\psi_1, \psi_2, \ldots, \psi_l)$ be event cue. The likelihood that the event cue $q$ corresponds to event $e$ is $L(q|e) = \prod_{i=1}^l \psi_i$.

The $\wedge$ symbol denotes logical AND operator so that when $q$ and $e$ are binary tuples, the cue $q$ matches event $e$ if $L(q|e) = 1$. Another version of Fuzzy AND operator will be introduced later in this paper which deals with real values between 0 and 1. We can also define $q_i$ and $e_i$ respectively as event and cue occur at relative time $t$. An event cue can be considered to match an event if the event has the maximum likelihood given the cue.

On the other hand, an episode can be defined as a sequence of events.

Definition 3. An episode $E$ is a sequence of events or $E = \{e_0, e_1, \ldots, e_n\}$. For event $e_i$, index $t$ indicates the relative time point the event occurs.

Matching an episode cue with a stored episode is not as straightforward as matching the event cue. The order dependency between events must be taken into account. In this case an episode can also be considered as a tuple or high-dimensional matrix representing the complete episode.

In probabilistic approaches, the pattern of sequential relations is usually maintained as transition probabilities between states or observation. For example, in HMM (Hidden Markov Model), if we can consider cues with observations and events as states, the transition probability $e_{ij}$ can be defined as $e_{ij} = P(e_{i+1} = e_j | e_i = e_i)$, for $0 \leq i, j \leq n$. Given that episode $E = \{e_0, e_1, \ldots, e_n\}$ and episode cue $Q = \{q_0, q_1, \ldots, q_l\}$. The probability that $E$ is the best match episode for cue $Q$ can roughly be defined as follows:

$$P(E|Q) = L(q_{max}) \prod_{i=0}^{n} \left[ P(e_i \mid e_{i-1}) \right] \mid q_{i}, q_{i+1} \ldots q_{n+1} \} \sum_{e_{max}}$$

(1)

This approach requires recursive processing to calculate the probability of every subsequent in the episode. The computational (time) complexity of the retrieval process using the probabilistic approach above is $O(m^n)$ where $m$ is the number of possible event (the event cue likelihood is omitted) and $n$ is the length of the episode cue presented. The transitional probability

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makes its space complexity to be $O(m^2N)$ where $N$ is the maximum length of episode possible.

This approach of sequential memory does not represent explicitly the episodes or sequences but only maintains the probability distribution over sequential paths. To retrieve an episode, the distribution must be calculated on-the-fly for every step of the sequence before retraicing the results to reconstruct the complete sequence.

This approach is still incapable to match events when the cues do not follow the exact order in the episode but still relatively or qualitatively so. For example, an episode cue $Q = [a, e]$ may be presented to retrieve the target episode $E = [a, b, c, d, e]$. Even though events in the cue may not be in the exact same position as the corresponding ones in the target episode, we still can say that the corresponding events in $Q$ have the same qualitative order as the ones in $E$ since $e$ is after $a$ in both $E$ and $E'$. This kind of cues is common in inferences and reasoning processes like planning (retrieving the planned sequence based on precondition and goals) and transitive inference. Similarly, the episode cue can be a number of consecutive events that may be located at the start, middle, the end of the sequence, or perhaps placed in any arbitrary position but still preserving their qualitative order within the target. To realize this kind of flexibility in probabilistic manner can be computationally impractical since the algorithm needs not just computing the distribution based on direct consecutive events but taking into account possible path through any possible event for each step in the sequence. Later in this section, a neural architecture is presented that resolves this qualitative order issues using its inherent and efficient pattern matching operations.

In what follows, the proposed neural architecture to support the functionality of episodic memory is described. The architecture supports approximate matching as well which is adjustable to relax the matching criteria as mentioned in the containment specification above. This adjustable approximate matching provides flexibility and more practical model but computationally efficient method of episodic memory.

3.2. Neural building block

Before presenting the complete proposed neural architecture, the basic building block of our model is described which is based on fusion ART [45] neural network. It can be viewed as an Adaptive Resonance Theory (ART) neural network [46] with multiple input (or output) fields. It supports various of learning paradigms to recognize and learn an incoming stream of input patterns across multiple channels in real time. Although it may not correspond directly to the real neural structure of hippocampus, fusion ART supports continual processing of information in cycles of categorizing, matching, learning, and dynamically allocating new neurons corresponding to many functional aspects of episodic memory.

Specifically, the fusion ART has $n$ input (output) field(s) and a single category field (Fig. 2(1)). An input (output) field consists of nodes or neurons representing a set of values.

**Definition 4.** Suppose $F_k$ and $F_s$ are the $k$th input (output) field and the category field of fusion ART respectively for $k = 1, \ldots, n$. Let $x^k$ denote the $F_k$ activity vector and $w^k$ denote the weight vector associating $k$th field with $j$th node in $F_s$. $F_k$ is associated with choice parameter $\alpha^k > 0$, learning rate $\beta^k \in [0, 1]$, contribution parameter $\gamma^k \in [0, 1]$, and vigilance parameter $\rho^k \in [0, 1]$.

All nodes in an input field $k$ are associated with every node in the category field by weighted connections. A node in the category field is selected through the interplay of two complementary processes: bottom-up selection and top-down matching. The node selected must meet the resonance criteria.

**Definition 5.** Vector $w^k_j$ relates every node of field $k$ with a node $j$ in the category field. Fuzzy AND ($\land$) and fuzzy OR ($\lor$) operations are defined by $p \land q = \min(p, q)$ and $p \lor q = \max(p, q)$ respectively. The norm $|v|$ is defined by $|v| = \sum_i |p_i|$ for vectors $v$ and $v^k$,

$$T_j = \sum_{k=1}^n |x^k \land w^k_j|,$$  

**Template matching** $m^k_j$ is a top-down matching value between category $j$ and the input $x^k$,

$$m^k_j = \frac{|x^k \land w^k_j|}{|x^k|},$$  

Based on the **resonance condition**, a node $j$ of category field can be selected if and only if

$$T_j = \max_j \left( T_j - m^k_j \geq \rho^k \right) \quad \text{for all } j \text{ th node in } F_s \text{ and } k \text{ th input field}.$$  

For each input field $k$, $\gamma^k \in [0, 1]$ regulates the node activation during the bottom-up activation and $\alpha^k > 0$ avoids division by zero. The vigilance parameter $\rho^k \in [0, 1]$ sets the level of tolerance for value differences between input pattern in $x^k$ and the stored template in $w^k_j$ during the top-down matching process.

Once the maximum element $j$ is selected, the corresponding patterns in $w^k_j$ can be updated based on the pattern in the input field. However, if no resonance exists, an uncommitted node associated with all 1s connection weights can be recruited and allocated as the selected node and is subjected to the template learning process. This feature of dynamic allocations of uncommitted node whenever the input pattern does not match makes the learning process incremental. The number of categories that can be learnt by the network can be limitless and does not have to be prescribed at the beginning. It is a remeniscence of the neurogenesis and plasticity features in hippocampus.

**Definition 6. Template learning** modifies weights associated with category $j$

$$w^{k\text{new}}_j = (1 - \rho^k) w^{k\text{old}}_j + \beta^k (x^k \land w^{k\text{old}}_j)$$  

The Learning rate parameter $\beta^k \in [0, 1]$ sets the rate of the connections change during learning. The $\land$ operation automatically generalizes the stored template whenever there is a difference between the current input and its weighted connections. Based on the selected node $j$, the pattern stored as weighted connections can be readout to a certain input (output) field $F_k^*$ such that $x^{k\text{new}} = w^*_j$.

Each entry in memory can be made to generalize similar inputs into the same category rather than as separate entries. This can be achieved by lowering the vigilance parameter $\rho^k$ so that slightly different input patterns will still activate the same category. Increasing vigilance makes the system sensitive to differences and learns specific information. On the other hand, lowering vigilance, keeps generalization to go on wherein small differences would be generalized to an existing similar category. This vigilance parameter can be considered as an analog of neuromodulators in the biological brain in which the release of them may influence the work of a group of nearby neurons.

Eqs. (2)-(5) are based on Fuzzy ART model [47]. Fuzzy ART simplifies the computation by replacing multiplication and addition operations respectively with Fuzzy AND and OR. Another
3.3. Generalization and learning abstract knowledge

From the definition of input (output) fields above, it is straightforward to express event or query as an input (output) of fusion ART. Event attribute \( v^e \in \{ e \} \) can be encoded as an input vector \( \mathbf{x}^e \) in fusion ART. The likelihood \( L \) can be stored as weighted connections \( \mathbf{w}_j^e \) between \( F_1 \) and \( F_2 \) field. Each node \( j \) in \( F_2 \) represents a single event so that the values of vector \( y \) can also correspond to the probability distribution of events given the input (cue).

Since some values of the attribute may not be specified (e.g., partial pattern in memory cue), it is necessary to make the input (output) structure incorporates non-specified or generalized values. This can be realized using complement coding.

Definition 7. The complement of vector \( \mathbf{v}^c \) denoted as \( \mathbf{v}^c \) is a vector consisting of the complements of each corresponding member of \( \mathbf{v} \) such that \( v^c_i = 1 - v^c_i \) where \( v^c_i \) and \( v^c_i \) are the ith items of \( \mathbf{v} \) and \( \mathbf{v} \) respectively. Complement coding of \( \mathbf{v} \) is the augmentation with its complement or \( \mathbf{v} \cup (\mathbf{v}^c) \).

To apply complement coding, input vector \( \mathbf{x}^e \) can be made as a complement coding of the event attribute \( \mathbf{v}^e \). The complement coding allows a value \( v^c \) to be generalized or unspecified by making \( v^c_1 = 1 - v_1 \) as the items of \( \mathbf{x}^c \).

Lemma 1. In fusion ART, if a value \( v^c \) specified in \( \mathbf{x}^c \) is provided such that \( v^c_1 = 1 - v^c_1 \) while the rest of values of \( \mathbf{x}^c \) are identical with weight vector \( \mathbf{w}^c \) and so the other corresponding fields and connections to node \( j \), then the network is still met with the resonance condition for node \( j \).

The proof will be provided in the appendix.

Complement coding enables a memory cue to be expressed without the need to specify all values so that a cue can be made partial to retrieve the complete one from memory. For example, if the event is a triple \( e = (1, 0, 1) \), a cue \( q = (1, -1, 1) \) can still match \( e \). Here, the second item of \( q \) is unspecified. Similarly, it is also possible to express not just a single real value of the attribute but a range of values. For example, 0.6 can be assigned as a value of attribute \( a \). It is also possible to assign a range so that \( a \) can be between 0.3 and 0.6. This is possible with complement coding and enables value generalization in learning. With complement coding, we also distinguish the actual input event as tuples from the input values to the neural network as vectors.

Besides making cues less specified by complement coding, generalization can also be learnt more permanently by the mechanism as described in Definition 6. Whenever a node \( j \) in \( F_2 \) field is selected, the template learning algorithm will make any stored value different from the corresponding value of the input vector to be less specified. In this case, there can be a stored value \( w_1^e \) in \( \mathbf{x}^e \) that \( w_1^e = 1 - w_1^e \). The rate of change towards generalization as in Eq. (5) depends on the \( \beta^e \) parameter.

This complement coding allows likelihood measure or pattern matching to be conducted for events and cues with different lengths of description. For instance, if \( |x| \) denotes the length of tuple \( x \), the likelihood \( L(q) \) of cue \( q \) to event \( e \) can still be measured even though \( |q| < |e| \) since some elements of \( q \) may be unspecified.

In this paper, it is regarded that this kind of generalization by learning is ubiquitous in the brain particularly in cortical areas. Consistent with the complementary concept of corticocortical model [28, 6], memories in that areas (e.g., semantic memory, procedural memory) tend to acquire general facts, routines, and abstract knowledge. Forming abstract knowledge may comprise gradual and iterative processes exposed with similar input patterns. In contrast, it is suggested in this paper that episodic memory tends to capture specific information into traversable forms as will be explained in detail later.

A single-layer fusion ART network can also be used to realize a memory system that stores events in a sequential order. Fig. 3(i) is a fusion ART model with two input (output) fields comparable to the probabilistic model (like HMM) of sequential memory described in Eq. (1). Here, the observation (or event cue) input is provided to \( F_1 \) field to get the likelihood of each event. For every event, the distribution of events at the next step is accumulated in \( F_2 \) to emulate the recursive function of Eq. (1).

Retrieving a particular segment of sequences from a semantic memory made as a network shown in Fig. 3(i) may be too complex since many non-standard calculations of fusion ART are involved. It is also possible to simplify the matter by relaxing the criteria of sequence retrieval to be a gradient following process similar to...
that in a reinforcement learning (RL) algorithm in which the state
is associated with actions and rewards. Here, the gradient can be
based on the rewards received during the learning shown in
Fig. 3(ii).

3.4. Encoding in episodic memory

As mentioned above, retrieving a particular segment of
sequences from a single layer fusion ART can be too complex
and may involve additional computations to calculate maximum
probabilities and retrace the results. Rather than maintaining
transition probabilities to represent the order, the activation values
of selected nodes are varied over time forming a profile of the
underlying sequence. The sequence profile is then learnt in
another layer explicitly as an episode. To retrieve an episode, a
similarity matching or resonance search can be employed between
the sequence profile of the cue and the target episode. The
additional ART layer gets the sequence profile as input from F2
field. The profile can be formed by maintaining and updating
values based on activation update function.

Definition 8. Function δ is activation update function where
∞ 0 ≤ δ ≤ 1. If yi is the jth node activation value at time t, then
its next activation value at time t + 1 is based on the update function
δ such that yi+1 = δ(yi).

By applying δ function to every element of vector y in F2, the
sequence profile will be formed in y. δ function can be specified as a
decay function if for every node value yi ∈ [y], yj ≥ δ(yj). The
sequence profile can be formed such that yj ≥ yj+1 ≥ yj+2 ≥ ...
≥ yj+n, or the node selected earlier will have a smaller or equal value
than the later one.

Lemma 2. If δ is a decay function and ynew ← δ(yold) is applied
iteratively for every element of vector y, then yj ≥ yj+1 ≥ yj+2 ≥ ...
≥ yj+n, will be formed in y where j is the node index selected at
time t.

The proof will be provided in the appendix.

Similarly, an increasing order of activation values can be
formed by making a growing (inverse decay) δ function such that
yj ≤ δ(yj). δ function determines how sequential pattern formed in
the presentation of the episode. The most straightforward way is a
linear decay function which creates a linear profile. It is also
possible to employ a non-linear one in which the decay function
amplifies recency and distant effects of retrieval accuracy will as
will be used in the case study in this paper.

However, the standard sequence profiling above still cannot
capture repetition. A selected node will be reselected when an
event repetition occurs within the same episode. In this case the
order of the event in the past is missing and becoming inconstant.
For instance, if two events e1 and e2 exist in an episode
whenever i ≠ k but e1 = e2, The profile matching based on the
decay function above may miss the event occurring earlier. When
i < k, the event e2 will be captured in vector y replacing the same
element yj, which is used to capture the earlier e1. The information
of the relative sequential position of the same but earlier event is
missing. Therefore, repetitions cannot be captured by the standard
approach of the decaying model.

To address this issue, the intrinsic feature of dynamically
allocating uncommitted node in ART is used. Every time the
currently activated node is reselected, a new uncommitted
node is allocated to capture the event instead of replacing the activation
of the node for the same earlier event. Let yi be the element of y
that has been selected to capture event ei. When the same event e1
occurs later within the same episode and matches with the stored
event represented by yi, the repetition can be identified since
yj > 0. In this case, a new element yh is allocated to represent
event e1 without replacing the sequential information in yj. In
other words, repetitions in this episodic memory are handled by
duplication.

Fig. 2(iii) shows the neural architecture of episodic memory
consisting of two layers of fusion ART. Events are processed and
stored between F2 and F3 fields when episodes are stored between
F2 and F3. Once the sequence profile in F2 is formed, it can be learnt
as a new episode by the additional layer (F3). Based on the bi-
directional activation and matching process in ART, a category I
representing an episode is selected in F2 such that

\[ T_i \cdot \begin{bmatrix} y \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \end{bmatrix} m \rightarrow \begin{bmatrix} y \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \cdot \gamma \cdot w_i \end{bmatrix} \gamma \cdot w_i \cdot T \cdot \max(c) \text{ for all F2 node i.} \]

(6)

Parameters α, γ, and the field index k are omitted as the upper
network only has a single input field (F2). Given the selected category I,
learning takes place such that wnew = (1 - β)wold + β(y * wold). ρ2 and β2 are the vigilance and learning rate parameters respectively of the field F2.

3.5. Episodic memory retrieval

The two-layers structure of fusion ART gives a significant
reduction to the space complexity for storing and processing
sequential representation. The space complexity of this episodic
memory model is O(mf) where m is the number of possible event
and f is the number of stored episodes (sequence). In comparison
to the aforementioned probabilistic approach, the proposed model
is no longer dependent on the maximum length of possible
sequences but determined by the number of episodes learnt. In
contrat, the space complexity of the probabilistic approach
(HMM) is O(mN) wherein N is the maximum length of possible
sequences. Since episode length is no longer the constraint, the
episode cue can be defined more flexibly by setting up the right
profile for the cues to retrieve the correct episode.

The sequence profile formation based on decaying δ function
implies that an input episode or cue y is recognized to have
occurred previously if all events in y have occurred in a stored
episode w and they are still in the same relative order.

Theorem 1. In the proposed model of episodic memory with a
decaying δ function, a sequence profile y will match or be in resonance condition
to jth node in F2 with the profile w if every member (event) of y is also in w
and w < w where w, and w are the relative time point of event
yj ∈ [y] and w ∈ [w] respectively.

The proof will be provided in the appendix.

The theorem above implies that the cue can be made as a
partial sequence of the stored episode to correctly retrieve the
right items as long as the values of the corresponding elements in
the target episode are greater or equal than the ones in the cue. For
example, given a stored episode consisting of a sequence of event
[A, B, C, D, E] in episodic memory, the sequence can be correctly
retrieved using episode cues containing [A, B, C], [A, B, D], or [D, E] if
the values are set according to the conditions as mentioned above.

Different profiles for retrieving the same episode are illustrated in
Fig. 4. The episode cue can match the target episode at the beginning, at the end, or somewhere in between.

This feature of retrieval with flexible cues contrasts the
proposed approach and most existing sequential learning models.
Unlike other models that allow online and incremental learning
for sequential patterns (e.g. HTM model [36]), the episodic

1 As a fair comparison, the complexity regarding the input or observation is
omitted for both the fusion-ART-based and the probabilistic model.
memory model is not based on dynamic transition links between nodes selected at different times, but instead making use of different node activation values to indicate directly the relative positions of selected nodes in the whole sequence.

It is also possible to make the cue profile not as strictly following the subsequent order but can be far apart relative to the target episode. For example, a profile of events $[A, E]$ with the same relative order as in the target, can still be match with the correct target. As shown in Fig. 5(iii), any positions of items in the cue such as the cue $[B, D]$ can still match the target episode, as long as the order relation $t_q < t_o$ holds for every event $q$ in the cue and its corresponding item $r$ in the target episode as suggested in Theorem 1. Interesting aspects take place whenever vigilance $\rho_2 < 1$ since the cue values does not have to be exactly less than or equal the corresponding values in the target. A low vigilance enables the cue profile to be made as in Fig. 5(iii). By making $\rho_2 < 1$, the resonance condition becomes less strict and more dependent on the maximum choice function as defined in Eq. (6).

Besides suggesting the formation of flexible activation profiles for memory cues, Theorem 1 also implies a drawback of this approach that in some conditions the profile may incorrectly match the sequential order. It may happen when the cue profile $\mathbf{y}$ consists of values that $y_i < y_j$ but the matching episode instead consists of the corresponding items in which $w_{ij} > w_{jk}$ even though every matching items in cue $\mathbf{y}$ is less than or equal target $\mathbf{w}$ as specified in Theorem 1. This condition can still exist if the matching condition like in Theorem 1 is violated but a smaller vigilance is applied. Fig. 6(iii) illustrate the flaw in which cue consisting of pattern $\{C, D\}$ matches two stored episodes $\{C, D, E, B\}$ (Fig. 6(i)) and $\{D, C, B, A\}$ (Fig. 6(ii)). Both matching patterns have the same activation value since they have the same length (total sum of elements), and thus, have the same chance to be selected even though the $\{C, D, E, B\}$ pattern should be more preferable in terms of its order resemblance.

However, this intrinsic flaw to retrieve the correct episode can be regarded as a possible source of natural recency effects in long-term memory in which items memorized relatively earlier are more erroneous to retrieve than the later. There is an uneven tendency to remember some latest happenings than earliest ones. The above characteristics can also explain how these early items can still be recalled relatively better if paired with some sequentially distant items since the later memory trace is relatively easier to retrieve and thus provide a better context to get the correct episode.

The retrieval drawback above can be resolved by inferences or manipulation of cues during retrieval so that the most suitable cue for the target can be formed. Inerring from episodic memory can mean forming or manipulating memory cues in certain ways in order to obtain information supporting a task achievement or solutions to a problem. In this case, memory cues becomes problem specifications and constraints to find the answer. As declarative memory, episodic memory should be based on explicit criteria or control which may be produced by a deliberative process involving other parts of the brain.

One direct approach to deal with the recency issue is to gradually modify the cue and while searching for any matching episode. A simple way is to keep employing $\delta$ function iteratively without receiving any input or selecting any category so that the cue gradually scales down to the right proper values. This search method can be very robust and has been evaluated to perform quite well in sequential recognition tasks in [22,39]. In this paper, the decaying search to simulate the awareness in memory retrieval is also employed.

On the other hand, few studies in neuroscience have also found that in some conditions, the sequential pattern in hippocampus is

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**Fig. 4.** An episode cue can be expressed as a partial sequence of the target episode (i) as a subsequence; (ii) as a few events at the beginning; and (iii) as a few events at the end.

**Fig. 5.** For inferences, memory cues can be arranged such that (i) events are far apart; (ii) events in any location of the target episode; and (iii) events are slightly drifted from the target episode.

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replayed in reversed order based on the recently happened episode (e.g. [49]). In this case, the decaying search method is unnecessary to emulate the phenomena since the cue can be formed based on the recently occurring events. It is also straightforward to playback the sequence in reverse, from the node that has the highest activation to the least. In any case, the proposed model is suitable for both the normal forward retrieval and the reverse replay modes.

3.6. Learning traversable knowledge

In many realistic settings like in realtime navigation, video game playing, or even some rigidly controlled tasks in a laboratory, the memory tasks comprise continuous interactions between inferences and retrieval. For example, in navigation tasks, spatial memory is used to remember the current location of the subject’s self before learning new locations in the next stage of exploration. In a general non-spatial task, like transitive inference, retrieval from episodic memory may provide the context in which the item to learn can be related.

In our model, this contextual memory orientation can be conducted by merging the input events to the retrieved context. Depending on the kind of task, the context can be a similar or coinciding stored episode so that the new event can be oriented and learnt relative to the context. For example, if subsequent A and B events were previously learnt as an episode, a sequence of B followed by C in the same task can be used to retrieve the previous A and B sequence as the context. The event C can then be appended to the last position of the context producing a new sequence of A, B, and C even though each of them may be learnt separately at distant times. Fig. 7 illustrates the steps taken in encoding new events based on their past contexts.

As previously mentioned, episodic memory is viewed to rapidly capture and record specific information about real tasks or experiences. The captured information are encoded and stored such that they are traversable following a particular order. The items stored can be considered as spatio-temporal maps in which the agent can travel around or back and forth following particular order. This conforms with the important feature of auto-features in episodic memory [4]. Sometimes, it is also possible to construct new information or sequence profile that may be novel or not experienced previously like transitive inferences, finding new routines in navigation, identifying novel patterns etc. In this paper, the use of episodic memory is demonstrated for transitive inference. In fusion ART, capturing specific knowledge can be realized by setting a high vigilance (ρ) for matching such that most incoming information tend to be regarded novel and allocated as new entries.

In contrast, memories in other cortical areas like semantic memory or procedural memory capture abstract or generalized knowledge that do not directly relate to experiences. In fusion ART, the automatic generalization in learning can be realized by applying low vigilance so that most incoming information are considered recurrent of things experienced in the past. A sequential information can be recalled from this network (e.g. procedural memory) by iteratively activating the same network towards the intended conditions. Unlike episodic memory, this kind of memory is hardly retrieved by going back and forth or traversing the temporal order further forward to predict the future.

In this paper, it is considered that episodic memory and semantic/procedural memory work together side by side to realize fast, complete, and robust memory system. In what follows the complementary memory system to handle transitive inference mechanisms is demonstrated. The next section presents some case studies to evaluate our model. The task involving transitive inference and sequence recognition are explained as examples of how the memory model supports inferences.

4. Case study and experiment

As a proof-of-concept, the episodic memory model is implemented and applied to solve transitive inference problems demonstrating its use to convey realtime inferences and reasoning. An experiment is conducted to show that the model can solve the transitive inference task and can be configured to exhibit the characteristic of human and animal subjects doing the same task.

4.1. Transitive inference

Transitive inference is a classic logical problem of weighing something over another. For example, given that A weighs more than B and B weighs more than C, it can be inferred that A also weighs more than C. Our experiment follows most other studies about transitive inference in human and animal in which the subjects are trained on a sequence of overlapping pairs to discriminate (A-B-, B-C-, C-D-, D-E-) where ‘-’ refers to the rewarded and nonrewarded choices respectively. The subjects are then tested to choose from a novel pairs like BD or BE. In one experiment using rats, the consistent successful choices in the test pair BD indicate the existence of a transitive inference. The capability of correctly selecting AE is considered too trivial [19]. Experiments involving animals generally compare the inferential capacity of normal subjects and hippocampus lesioned ones.
4.2. The memory architecture

Following other simulated domain of transitive inference in episodic memory [50,8], a separated but connected memory system to episodic memory is built corresponding to a procedural memory in cortical area. A similar view is adopted in [50] that the cortical part is made for selecting a quick response based on rewarding state or condition.

A procedural memory is made to realize the cortical system for learning and decision making. A reinforcement learning (RL) algorithm is employed to a single fusion ART neural network as procedural memory running a direct-access procedure [51] with immediate reward. The memory is used to search the network using the resonance search process for a match with the current input state and retrieve the action.

Let \( x_1 ^t, x_2 ^t, \) and \( x_3 ^t \) be the input vector for the state, action, and reward respectively for the procedural memory fusion ART network. The vector structure of the input is shown in Fig. 9(i). Basically, it selects a category node based on the state vector (the action vector is set to all \( 1 (1, \ldots , 1) \) vector and the reward vector to the maximum value pair \( (1,0) \)). In this way, it searches for a node representing the current state that leads to the maximum reward. If a resonance node is found, the action vector is readout so that it can be selected for execution. The memory system receives a reward immediately after executing the action. The reward value can be 1 if the reward is obtained or 0 for no reward. \( x_3 ^t \) can be set either \((1,0)\) for positive reward or \((0,1)\) for the negative one. Based on the reward received, the action selected, and the previous state, a category can be learnt. The cycle continues by updating state vector according to the new case before moving on to the next cycle.

![Time order](image)

**Fig. 7.** A new episode can be stored and contextualized to other related episode; (i) the new episode activates profile in \( F_3 \) as a cue; (ii) the matching episode is recalled; and (iii) a new activation profile is created by merging.

![Procedural Memory](image)

**Fig. 8.** The memory architecture combining episodic memory with procedural memory.

Both procedural and episodic memory share the same input source and can learn independently.

The structure of the input fields is shown in Fig. 9(i). The input state is a binary vector corresponding to the presented item. Each item is presented one at a time. In our transitive domain, two actions must be chosen by the subject. The first action is to select the item just appears in the state field and the second one is to switch (ignoring the current state) or move on to the other item that appears next. For example, when A and B are presented consecutively, the select action chooses A directly when presented, but the switch action ignores it and waits until the presentation of B. The use of the two actions follows the settings in many different simulation models for transitive inference [50,8,37]. In the experiments of transitive inference using human and animals, two actions are also alike such as selecting the right or left item for human or digging sand for the reward in the first cup or the latter for rats.

The procedural memory learns the association between event state, action, the reward received after taking the last action. On the other hand, the episodic memory learns the item selection task by employing the context encoding process. The episodic memory receives a single item in the input state field at a time, considers the select or switch action, and immediately receives the reward similar to the procedural memory. However, each selection event relates to one another by their contextualized sequence profiles.

When episodic memory receives an event and cue in the input field, it retrieves an episode with the same context (switch or select; rewarding or non-rewarding state). If the exact match is found, the system just perform the readout action. If no identical
match is found, the event with state, action, and the resulting reward is appended to the existing episode with the same context letting the episode to grow. The activation update process is conducted after the event insertion as follows:

\[
\begin{align*}
\delta_j^{(new)}(t) &= \begin{cases} 
I & \text{if } j = I \text{ and } y_j^{(old)} \leq 0 \\
\delta \left(y_j^{(old)} \right) & \text{if } (1 - I) \leq y_j^{(old)} \leq I \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]  

where \( I \) is the maximum activation level in \( y \), \( 0 < I < 1 \). \( \delta \) function used in this model can be defined as follows:

\[
\delta(y) = y(1 - ry).
\]

\( r \in [0, 1] \) is the decay rate of the node activation. This decay function is particularly chosen for transitive learning task in the experiment to make the decaying factor gets smaller over time to retain the events in the beginning of the episode.

In the experiment, \( r = 0.7 \) is applied for the decaying parameter to form the cue with stochastically varying vigilance parameters. The \( r \) value corresponds to the immediate distance between two items in the sequence profile. Based on the cue, a matching episode can be found through the resonance search and the right action can be retrieved by searching the retrieved episode for an event consisting of the rewarded action as the context.

Both the procedural memory and episodic memory do not work individually in isolation. In a single trial they are working together exhibiting the memory strategy to choose the right action. Whenever an input event is received the steps followed by both memory systems can be described as follows:

1. Each input event is held in the state field of the procedural memory while the episodic memory retrieves the context based on the maintained profile including the previous event. The event cue consists of state, unspecified action, and the maximum possible reward received.
2. A perfect match in procedural memory leads to the performance of action retrieved in procedural memory.
3. A perfect match in episodic memory leads to the performance of action retrieved.
4. The reward feedback together with the previous state and selection are learnt by both procedural and episodic memory.

4.3. Experimental settings

There are six stimulus items to be discriminated by the subjects: A, B, C, D, E, and F. Following the procedure using human subjects in [17], there are two main stages in the experiment. Firstly, the subject is trained with five-problem discrimination set \( \{A \leftrightarrow B, B \leftrightarrow C, C \leftrightarrow D, D \leftrightarrow E, E \rightarrow F\} \) where \( "\leftrightarrow" \) and \( "\rightarrow" \) refer to the rewarded and non-rewarded choices, respectively. The training consists of four phases of blocked trials, followed by a randomly interleaved trials. In the first phase, the premise pairs to be discriminated are presented in blocks of six trials wherein the first block consists of AB trials (A+B-), the second block consists of BC trials (B+C-), the third block consists of CD trials (C+D-), and so on until the EF trials. In phase 2 the number of trials in each block is reduced to four trials. Phases 3 and 4 consist of only three and two trials respectively. These blocked trials are followed by 25 trials of randomly chosen premise pairs. Secondly, the subject is tested with four different novel combinations (BD, CE, BF, and AF). In the testing stage, all pairs are randomly presented similar to the last phase in the training stage, however the outcome of the selected action is no longer learnt by the memory system. Another testing stage is also employed to test the robust continuous decaying search procedure in the episodic memory to simulate the awareness in solving the task to compare with the human data in awareness condition.

All configuration and parameters in the memory system are set to the same parameter values for all trials in the learning stage. The testing stage applies the same set of parameters as in the training stage except the vigilance parameter \( \rho \) for \( F_2 \) field in the episodic memory which is varied for all testing trials. All fields in both cortical and episodic memory employs the same parameters \( \beta = 1, \gamma = 1, \alpha = 0.1 \). The vigilance parameters for all input fields of the episodic memory are set to 1. In the training stage, the \( F_2 \) vigilance of the episodic memory is high \((\rho_2 = 0.9)\) for retrieval, but lower \((\rho_2 = 0.4)\) for the learning to ensure that events in the same context can be united in a single episode.

A similar set of parameters is also applied to the network but the vigilance for the state, action, and reward field are set to 0.6, 0, and 1 respectively. In the testing stage, \( \rho_1 \) is randomly set for each trial based on a Gaussian distribution with the center point \( \mu = 0.65 \) and three different standard distribution \( \sigma = 0.1, \sigma = 0.2, \sigma = 0.35 \) to emulate the variation of individual subject. This variation of gaussian distribution is employed based on the conjecture that the characteristics of recency effect and the lag-in-pair dependency are due to the variation of consistency in tolerating different patterns by each individual. Less varied tolerance or more consistent pattern of choices improves the accuracy of retrieving from memory.

4.4. Results

After the training stage, the weighted connections in the episodic memory form two almost symmetrical patterns of associative strength between the two contexts learnt as two different episodes. Fig. 5(ii) shows the weight values after the training stage. The weights pattern formed in the episodic memory shows that the contextualization in the encoding process can group different events together although they are presented separately in different trial blocks in the testing stage. The low vigilance \((\rho_2 = 0.4)\)
used in the episode learning enables the network to group the
contexts into two episodes.

To compare with the results from human subjects, we take the
average from 57 different testing runs. This corresponds to 57
human subjects tested in the unaware transitive inference task
[17]. Fig. 10 shows the comparison of percent corrects in the
unaware tasks. The figure shows that the instantaneous transitive
performance in our episodic memory model is consistent with
human performance unaware of the sequential relations prior to
the test. There is a dependency of the distance between item pairs
in the sequence (e.g. BE is discriminated better than BD). However,
it is indicated that the recency of the items in the sequence also
contributes to the task performance (e.g. CE which comes latter in
the sequence can be discriminated better than CD although both
have the same sequential distance between the items). The
performance characteristics are also consistent with the results from
the cortical/hippocampal (CH) model of episodic memory [50].
However, our model does not need a separate stimulus selection
module to produce the effect.

The procedural memory is also tested to work in isolation after
the learning without the episodic memory. This condition simu-
lates a lesioned hippocampus commonly investigated in many
studies of hippocampus. The test shows that for item pairs located
internally in the sequence (BD, CE, and BE), the performance is not
better than the chance level. However, the performance is much
better for the pair with items at the beginning and the end of the
sequence (anchor points). The results are consistent with the
prediction in the CH model [50] in which the superior perfor-
mance of the anchor points are caused by the absence of rewards
for the item at the end of the sequence (F) while the starting item
(A) is always rewarded. The relatively low vigilance ($\psi_1 = 0.6$) of
the state field in procedural memory is enough to match the input
with either items. Different $\sigma$ produces different performance and
variations though the tendency is sustained. The highest $\sigma$ creates
more variation but also less performance. The lower $\sigma$ produces
less variation with higher performance for every pair combination.

Despite the performance characteristics caused by the item
positions in the sequence, a way to overcome the constraint is also
suggested in this paper. The continuous search process to improve
the performance of retrieving the correct response in the transitive
inference is also tested. Fig. 10(ii) shows the performance of the
episodic memory model with continuous search process com-
pared with the human performance. Consistent with the human
data with awareness, the results show that almost all test pairs can
be solved correctly. A small error at the earlier pairs (BD) is caused
by the random gaussian level of vigilance still applied in the test
configuration. The results show that the model can be flexibly
configured to robustly solve sequential tasks like the transitive
inference by transforming the memory cues during retrieval. It is
also shown that smaller $\sigma$ causes better performance.

5. Conclusion

We have presented the neural model of episodic memory as a
long-term temporary buffer for rapidly storing and recalling
episodes supporting inferences and decision making. The model is
made as two layers of multiple channels adaptive resonance
theory (ART) neural network structure exploiting a special neural
encoding wherein the neurons’ activation values and the synaptic
weights are proportional to the relative sequential order in which
the corresponding neurons are fired. This encoding technique can
be used to explain many characteristics of sequential processing in
memory. The model conforms with the evidences involving
humans and animals, in which the locations of memory items
within the sequence influence the accuracy of memory retrieval.
Distant items are easier to retrieve than the closer ones. On the
other hand, recently accessed or memorized items are easier to
retrieve.

We suggest that the proposed model offers significant reduc-
tion to the space complexity to store sequences or episodes
compared to other common types of sequential memory like
probabilistic, Markovian, or recurrent network models. This reduc-
tion is possible since there is no need to maintain the transitions
between events explicitly to represent the order. Instead, the real
values of the selected events over time are sufficient to profile the
relative sequential order in the episode.

We have also proposed a new encoding mechanism that stores
and links events into its appropriate context. The contextualiza-
tion is conducted by merging a newly encountered event into a similar
or coinciding past episode. By orienting the event according to its
past context, the memory maps the task environment. We suggest
that this approach of encoding enables the episodic memory to
support conjunctive representation for spatial, temporal, or con-
textual processing beyond simply sequential relations.

Some formulation and mathematical analysis has been pre-
sented to disclose the characteristics and robustness of the
proposed model. It is clearly shown that different types of memory
cues to retrieve episodes can be formed without arranging them
strictly in the right subsequent order. This feature can make the
retrieval accuracy robust and quite resistant to partiality and
noises in memory cues. The robustness in retrieval accuracy is
made possible by the inherent bi-directional search towards the
best match in the ART building block of the episodic memory
model. The characteristics and features of the proposed model
allow inference and reasoning to be based on episodic memory by

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forming or manipulating memory cues. The cues become task specifications or constraints to the solutions. An important theorem as the basis of the cues partiality and flexibility is presented that partial or incomplete cues can always match the target episode as long as their matching events relatively occur at or before the corresponding events in the target.

Despite the promising results, many aspects of episodic memory are still not fully explored and investigated in our model. More generic and efficient contextualization process for encoding is desirable to be able to deal with more general domain and task environment beyond transitive inferences. In the future we shall apply our model to learn and solve more complex tasks. For example, the model can be applied to a path-finding problem in a navigation task or extended to a more generic planning domain. We may also investigate the potential of our model as an embedded part of a computational agent architecture or application. We may extend the model to investigate more realistic model of reasoning and learning like different types of inference, reverse playback phenomena, influence of neuromodulators and other more complex reasoning processes and mechanisms.

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Appendix A. Proofs

A.1. Proof for Lemma 1

In fusion ART, if a value \( v_4 \) specified in \( x^4 \) is provided such that \( v_4(\ast) \leq 1 - P_{v_4} \) while the rest of values of \( x^4 \) are identical with weight vector \( w^4 \) and so the other corresponding fields and connections to node \( j \), then the network is still met with the resonance condition for node \( j \).

Proof. Let \( (v_4, P_{v_4}) \) and \( (w_4, P_{w_4}) \) be the pair of complemented values in input \( x^4 \) and its corresponding pair of weight vectors in \( w^4 \) respectively. It is clear that if \( v_4 \leq w_4 \leq 1 - P_{v_4} \) and \( w_4 = 1 - P_{w_4} \), then \( v_4 \leq w_4 \leq P_{v_4} \) and \( P_{w_4} \leq P_{v_4} \). Therefore, \( (v_4, P_{v_4}) \leq (w_4, P_{w_4}) \leq (v_4, P_{v_4}) \). As the rest of fields are identical with the corresponding connections, the template matching \( m_j = \sum_1^{n_k} w_j = \sum_1^{n_k} v_j = 1 \). Since the other fields are identical with the corresponding weight vectors, the network is in resonance condition.

A.2. Proof for Lemma 2

If \( \delta \) is a decay function and \( y^{\text{new}} = \delta y^{(M)} \) is applied iteratively for every \( i \)-th element of vector \( y \), then \( y_{1} \geq y_{i} \geq y_{i} \geq \ldots \geq y_{j} \) will be formed in \( y \) where \( j \) is the index selected at time \( t \).

Proof. Let \( y_{j} \) be the activation value of a node selected at time \( t \). It can be assumed that every selected element has the same maximum value at the time it is selected or \( y_{j} = y_{j} \geq y_{j} \geq \ldots \geq y_{j} \). Since \( y_{j} = \delta y_{j} \) and \( y_{j} = \delta y_{j} \), consequently \( y_{j} 

\begin{align*}
A.3. Proof for Theorem 1

In the proposed model of episodic memory with a decaying \( \delta \) function, a sequence profile \( y \) will match or be in resonance condition to \( j \)-th node in \( F \) with the profile \( w \) if every member (event) of \( y \) is also in \( w \) and \( y_{j} \) and \( w_{j} \) are the relative time point of event \( y_{j} \) and \( w_{j} \) respectively.

Proof. Given that both profile \( y \) and \( w \) are vectors, \( y_{j} \leq w_{j} \) implies that \( y_{j} = w_{j} \). Since for every element \( y_{j} \) and \( w_{j} > 0 \), the corresponding \( j \)-th element of \( w \) also holds that \( w_{j} > 0 \). It is obvious that \( y_{j} \leq w_{j} \). Based on Eq. (6), \( m_j = \sum_1^{n_k} w_j = 1 \) or \( y_{j} \) is always in resonance condition to \( j \). By contradiction, if \( y_{j} \) is not in resonance condition, then \( y_{j} > m_j \). However, this contradicts \( y_{j} = 1 \) which concludes that \( y_{j} \) must be in resonance condition to episode \( j \).

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